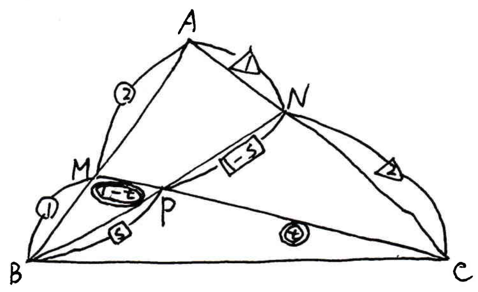
平面ベクトル(1)(2022年阪大)



(1) $BP : PN = s : (1-s)$, $CP : PM = t : (1-t)$とすると.

$\overrightarrow{AP} = (1-s)\overrightarrow{AB} + s\overrightarrow{AN} = (1-s)\overrightarrow{AB} + s \cdot \frac{1}{3}\overrightarrow{AC}$ ―①

$\overrightarrow{AP} = (1-t)\overrightarrow{AC} + t\overrightarrow{AM} = (1-t)\overrightarrow{AC} + t \cdot \frac{2}{3}\overrightarrow{AB}$ ―②

$\overrightarrow{AB} \neq \vec{0}$, $\overrightarrow{AC} \neq \vec{0}$, $\overrightarrow{AB} \not\parallel \overrightarrow{AC}$だから. ①, ②より. $1-s = \frac{2}{3}t$ ―③ $\frac{1}{3}s = 1-t$ ―④

③より. $s = 1 - \frac{2}{3}t$ ―③′ ③′, ④より. $\frac{1}{3} - \frac{2}{9}t = 1-t$ $\frac{7}{9}t = \frac{2}{3}$ $t = \frac{2}{3} \cdot \frac{9}{7} = \frac{6}{7}$

$\therefore \overrightarrow{AP} = \frac{6}{7} \cdot \frac{2}{3}\overrightarrow{AB} + \left(1 - \frac{6}{7}\right)\overrightarrow{AC} = \frac{4}{7}\overrightarrow{AB} + \frac{1}{7}\overrightarrow{AC}$

(2) $|\overrightarrow{BC}| = a$だから. $a^2 = |\overrightarrow{BC}|^2 = |\overrightarrow{AC} - \overrightarrow{AB}|^2 = |\overrightarrow{AC}|^2 - 2\overrightarrow{AB} \cdot \overrightarrow{AC} + |\overrightarrow{AB}|^2$

また. $|\overrightarrow{AC}| = b$, $|\overrightarrow{AB}| = c$だから. $a^2 = b^2 - 2\overrightarrow{AB} \cdot \overrightarrow{AC} + c^2$より. $\overrightarrow{AB} \cdot \overrightarrow{AC} = \frac{1}{2}(b^2 + c^2 - a^2)$

これと(1)より. $|\overrightarrow{AP}|^2 = \left|\frac{1}{7}(4\overrightarrow{AB} + \overrightarrow{AC})\right|^2 = \frac{1}{7^2}(16|\overrightarrow{AB}|^2 + 8\overrightarrow{AB} \cdot \overrightarrow{AC} + |\overrightarrow{AC}|^2)$

$= \frac{1}{7^2}(16c^2 + 4b^2 + 4c^2 - 4a^2 + b^2) = \frac{1}{7^2}(-4a^2 + 5b^2 + 20c^2)$

$\therefore AP = |\overrightarrow{AP}| = \frac{1}{7}\sqrt{-4a^2 + 5b^2 + 20c^2}$

(1)別解 メネラウスの定理より. $\frac{AB}{BM} \cdot \frac{MP}{PC} \cdot \frac{CN}{NA} = 1$だから. $\frac{3}{1} \cdot \frac{MP}{PC} \cdot \frac{2}{1} = 1$より $\frac{MP}{PC} = \frac{1}{6}$

よって. $MP : PC = 1 : 6$だから.

$\overrightarrow{AP} = \frac{6\overrightarrow{AM} + \overrightarrow{AC}}{7} = \frac{1}{7}\left(6 \cdot \frac{2}{3}\overrightarrow{AB} + \overrightarrow{AC}\right) = \frac{4}{7}\overrightarrow{AB} + \frac{1}{7}\overrightarrow{AC}$